Lebanese University
Faculty of information
Section 1

Final Exam: Discrete Mathematics

Time: 90 minutes
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Problem 1.

1. Show that these conditional statements are logically equivalent.
   a) $\neg(p \leftrightarrow q)$ and $p \leftrightarrow \neg q$
   b) $(p \to q) \vee (p \to r)$ and $p \to (q \vee r)$

2. Show that each of these conditional statements is a tautology by using truth tables.
   a) $[\neg p \wedge (p \vee q)] \to q$
   b) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

Problem 2.

1. Suppose A, B and C are sets. Show that:

$$(A - B) \subseteq A \cap \bar{B}$$

2. Show that if A, B and C are sets then $\overline{A \cup B} = \bar{A} \cap \bar{B}$.
   a) By showing each side is a subset of the other side.
   b) Using a membership table.

Problem 3.

1.
   a) Find an inverse of a modulo m for each of these pairs of relatively prime integers.

$$a = 6, m = 77.$$

   b) Solve each of these congruences using the modular inverses found in part (a)

$$6x \equiv 5 \pmod{77}$$

Problem 4.

1. Let R the relation defined on $\mathbb{R}$ by:

$$xRy \Leftrightarrow x^2 - y^2 = x - y$$

   Show that the relation R is an equivalence relation on the set of integers.

2. Determine the equivalence class of x for any real x.

3. Let R_1 and R_2 be relations on a set A represented by the matrices

$$M_{R_1} = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad M_{R_2} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

   Find the matrix representing
   a) $R_1 \cup R_2$ and $R_1 \cap R_2$

b) $R_1 \circ R_2$

c) $R_1 \oplus R_2$

d) $\overline{R_2}$ and R_1^{-1}

Good Luck